OCR RECOGNISING ACHIEVEMENT

**ADVANCED GCE** **4730/01**

**MATHEMATICS**

Mechanics 3

**THURSDAY 17 JANUARY 2008** Afternoon

Time: 1 hour 30 minutes

**Additional materials:** Answer Booklet (8 pages)
List of Formulae (MF1)

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

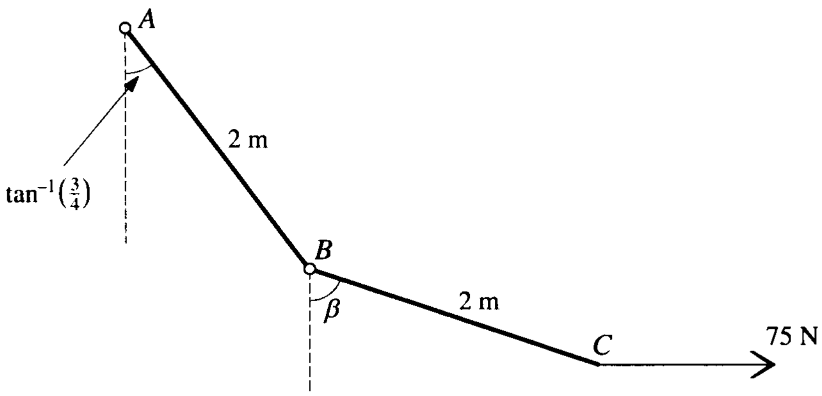
1 A smooth horizontal surface lies in the $x$-$y$ plane. A particle $P$ of mass 0.5 kg is moving on the surface with speed $5\text{ m s}^{-1}$ in the $x$-direction when it is struck by a horizontal blow whose impulse has components $-3.5$ N s and 2.4 N s in the $x$-direction and $y$-direction respectively.

(i) Find the components in the $x$-direction and the $y$-direction of the velocity of $P$ immediately after the blow. Hence show that the speed of $P$ immediately after the blow is $5.2\text{ m s}^{-1}$. [4]

$P$ is struck by a second horizontal blow whose impulse is $\mathbf{I}$.

(ii) Given that $P$'s direction of motion immediately after this blow is parallel to the $x$-axis, write down the component of $\mathbf{I}$ in the $y$-direction. [2]

2



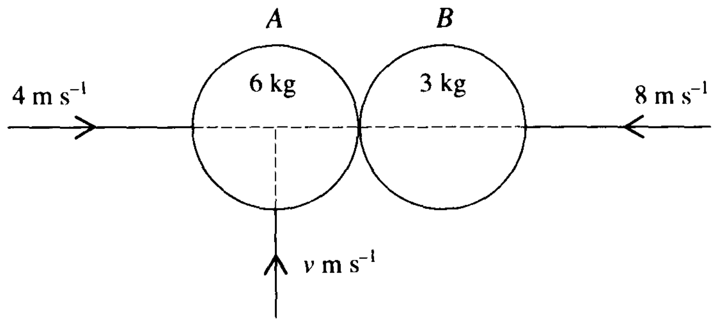

Two uniform rods $AB$ and $BC$, each of length 2 m, are freely jointed at $B$. The weights of the rods are $W$ N and 50 N respectively. The end $A$ of $AB$ is hinged at a fixed point. The rods $AB$ and $BC$ make angles $\tan^{-1}\left(\frac{3}{4}\right)$ and $\beta$ respectively with the downward vertical, and are held in equilibrium in a vertical plane by a horizontal force of magnitude 75 N acting at $C$ (see diagram).

(i) By taking moments about $B$ for $BC$, show that $\tan\beta = 3$. [3]

(ii) Write down the horizontal and vertical components of the force acting on $AB$ at $B$. [2]

(iii) Find the value of $W$. [4]

3




Two uniform smooth spheres $A$ and $B$, of equal radius, have masses 6 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision the velocity of $A$ has components $4\text{ m s}^{-1}$ along the line of centres towards $B$, and $v\text{ m s}^{-1}$ perpendicular to the line of centres. $B$ is moving with speed $8\text{ m s}^{-1}$ along the line of centres towards $A$ (see diagram). The coefficient of restitution between the spheres is $e$.

**(i)** Find, in terms of $e$, the component of the velocity of $A$ along the line of centres immediately after the collision. [5]

**(ii)** Given that the speeds of $A$ and $B$ are the same immediately after the collision, and that $3e^2 = 1$, find $v$. [4]

4 A particle of mass $m$ kg is released from rest at a fixed point $O$ and falls vertically. The particle is subject to an upward resisting force of magnitude $0.49mv$ N where $v\text{ m s}^{-1}$ is the velocity of the particle when it has fallen a distance of $x$ m from $O$.

**(i)** Write down a differential equation for the motion of the particle, and show that the equation can be written as $\left(\frac{20}{20-v} - 1\right)\frac{dv}{dx} = 0.49$. [5]

**(ii)** Hence find an expression for $x$ in terms of $v$. [5]

5 A particle $P$ of mass $m$ kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity $0.75mg$ N. The other end of the string is attached to a fixed point $O$ of a smooth plane inclined at 30° to the horizontal. $P$ is released from rest at $O$ and moves down the plane.

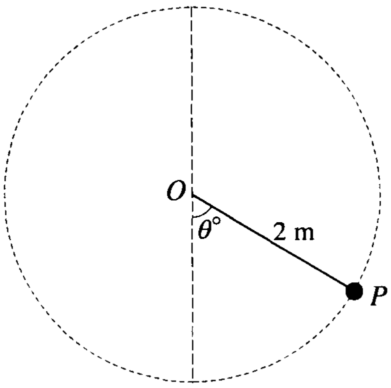
**(i)** Show that the maximum speed of $P$ is reached when the extension of the string is 0.8 m. [3]

**(ii)** Find the maximum speed of $P$. [4]

**(iii)** Find the maximum displacement of $P$ from $O$. [4]

**[Questions 6 and 7 are printed overleaf.]**

6




A particle $P$ of mass 0.4 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point $O$. With the string taut the particle is travelling in a circular path in a vertical plane. The angle between the string and the downward vertical is $\theta°$ (see diagram). When $\theta = 0$ the speed of $P$ is $7\ \text{m s}^{-1}$.

**(i)** At the instant when the string is horizontal, find the speed of $P$ and the tension in the string. [4]

**(ii)** At the instant when the string becomes slack, find the value of $\theta$. [8]

7 A particle $P$, of mass $m$ kg, is attached to one end of a light elastic string of natural length 3.2 m and modulus of elasticity $4mg$ N. The other end of the string is attached to a fixed point $A$. The particle is released from rest at a point 4.8 m vertically below $A$. At time $t$ s after $P$'s release $P$ is $(4 + x)$ m below $A$.

**(i)** Show that $4\frac{d^2x}{dt^2} = -49x$. [3]

$P$'s motion is simple harmonic.

**(ii)** Write down the amplitude of $P$'s motion and show that the string becomes slack instantaneously at intervals of approximately 1.8 s. [4]

A particle $Q$ is attached to one end of a light **inextensible** string of length $L$ m. The other end of the string is attached to a fixed point $B$. The particle is released from rest with the string taut and inclined at a small angle with the downward vertical. At time $t$ s after $Q$'s release $BQ$ makes an angle of $\theta$ radians with the downward vertical.

**(iii)** Show that $\frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta$. [3]

The period of the simple harmonic motion to which $Q$'s motion approximates is the same as the period of $P$'s motion.

**(iv)** Given that $\theta = 0.08$ when $t = 0$, find the speed of $Q$ when $t = 0.25$. [5]

# 4730 Mechanics 3

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|---|---|---|---|---|
| 1 | (i) $[0.5(v_x - 5) = -3.5,\ 0.5(v_y - 0) = 2.4]$ | M1 | | For using I = m(v – u) in x or y direction |
| | Component of velocity in x-direction is $-2\text{ms}^{-1}$ | A1 | | |
| | Component of velocity in y-direction is $4.8\text{ms}^{-1}$ | A1 | | |
| | Speed is $5.2\text{ms}^{-1}$ | A1 | 4 | AG |
| SR For candidates who obtain the speed without finding the required components of velocity (max 2/4) | | | | |
| | Components of momentum after impact are -1 and 2.4 Ns | B1 | | |
| | Hence magnitude of momentum is 2.6 Ns and required speed is $2.6/0.5 = 5.2\text{ms}^{-1}$ | B1 | | |
| | (ii) | M1 | | For using $I_y = m(0 - v_y)$ or $I_y$ = -y-component of 1st impulse |
| | Component is –2.4Ns | A1 | 2 | |

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|---|---|---|---|---|
| 2 | (i) | M1 | | For 2 term equation, each term representing a relevant moment |
| | $50\text{x}1\sin\beta = 75\text{x}2\cos\beta$ | A1 | | |
| | $\tan\beta = 3$ | A1 | 3 | AG |
| | (ii) Horizontal force is 75N | B1 | | |
| | Vertical force is 50N | B1 | 2 | |
| | (iii) | M1 | | For taking moments about A for the whole or for AB only |
| | For not more than one error in $\text{Wx}1\sin\alpha + 50(2\sin\alpha + 1\sin\beta) = 75(2\cos\alpha + 2\cos\beta)$ or $\text{Wx}1\sin\alpha + 50\text{x}2\sin\alpha = 75\text{x}2\cos\alpha$ | A1 | | Where $\tan\alpha = 0.75$ |
| | 0.6W + 107.4... = 167.4… or 0.6W + 60 = 120 | A1 | | |
| | W = 100 | A1 | 4 | |

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|---|---|---|---|---|
| 3 | (i) | M1 | | For using the principle of conservation of momentum in the **i** direction |
| | 6x4 – 3x8 = 6a + 3b (0 = 2a + b) | A1 | | |
| | | M1 | | For using NEL |
| | (4 + 8)e = b – a (12e = b – a) | A1 | | |
| | Component is $4e\text{ ms}^{-1}$ to the left | A1 | 5 | 'to the left' may be implied by a = -4e and arrow in diagram |
| | (ii) $b = 8e\text{ ms}^{-1}$ | B1ft | | ft b = -2a or b = a + 12e |
| | | M1 | | For using '**j** component of A's velocity remains unchanged' |
| | $(8e)^2 = (4e)^2 + v^2$ | A1ft | | ft $b^2 = a^2 + v^2$ |
| | v = 4 | A1 | 4 | |

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| 4 | (i) [mg – 0.49mv = ma] | M1 | | For using Newton's second law |
| | $mv\frac{dv}{dx} = mg - 0.49mv$ | A1 | | |
| | $\left[\frac{v(dv/dx)}{g - 0.49v} = 1\right]$ | M1 | | For relevant manipulation |
| | $\left[\frac{v}{9.8 - 0.49v} \equiv \frac{-1}{0.49}\left(\frac{(9.8 - 0.49v) - 9.8}{9.8 - 0.49v}\right)\right]$ | M1 | | For synthetic division of v by g - 0.49v, or equivalent |
| | $\left(\frac{20}{20 - v} - 1\right)\frac{dv}{dx} = 0.49$ | A1 | 5 | AG |
| | (ii) | M1 | | For separating the variables and integrating |
| | $\int \frac{20}{20 - v}dv = -20\ln(20 - v)$ | B1 | | |
| | -20 ln(20 – v) –v = 0.49x (+C) | A1ft | | |
| | [-20 ln20 = C] | M1 | | For using v = 0 when x = 0 |
| | x = 40.8(ln20 – ln(20 – v)) – 2.04v | A1 | 5 | Accept any correct form |

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| **5** | (i) | M1 | | For using Newton's second law with a = 0 |
| | $mg\sin30^o = 0.75mgx/1.2$ | A1 | | |
| | Extension is 0.8m | A1 | 3 | AG |
| | (ii) PE loss = $mg(1.2 + 0.8)\sin 30^o$ (mg) | B1 | | |
| | EE gain = $0.75mg(0.8)^2/(2x1.2)$ (0.2mg) | B1 | | |
| | [ $\frac{1}{2}mv^2 = mg - 0.2mg$] | M1 | | For an equation with terms representing PE, KE and EE in linear combination |
| | Maximum speed is $3.96ms^{-1}$ | A1 | 4 | |
| | (iii) PE loss = $mg(1.2 + x)\sin30^o$ or $mgd\sin30^o$ | B1ft | | ft with x or d – 1.2 replacing 0.8 in (ii) |
| | EE gain = $0.75mgx^2/(2x1.2)$ or $0.75mg(d - 1.2)^2/(2x1.2)$ | B1ft | | ft with x or d – 1.2 replacing 0.8 in (ii) |
| | [$x^2 - 1.6x - 1.92 = 0$, $d^2 - 4d + 1.44 = 0$] | M1 | | For using PE loss = EE gain to obtain a 3 term quadratic in x or d |
| | Displacement is 3.6m | A1 | 4 | |
| Alternative for parts (ii) and (iii) for candidates who use Newton's second law and a = v dv/dx: In the following x, y and z represent displacement from equil. pos$^n$, extension, and distance OP respectively. | | | | |
| | [$mv\,dv/dx = mg\sin30^o - 0.75mg(0.8 + x)/1.2$, $mv\,dv/dy = mg\sin30^o - 0.75mgy/1.2$, $mv\,dv/dz = mg\sin30^o - 0.75mg(z - 1.2)/1.2$] | M1 | | For using N2 with a = v dv/dx |
| | $v^2/2 = -5gx^2/16 + C$ or $v^2/2 = gy/2 - 5gy^2/16 + C$ or $v^2/2 = 5gz/4 - 5gz^2/16 + C$ | A1 | | |
| | [$C = 0.6g + 5g(-0.8)^2/16$ or $C = 0.6g$ or $C = 0.6g - 5g(1.2/4) + 5g(1.2)^2/16$ | M1 | | For using $v^2(-0.8)$ or $v^2(0)$ or $v^2(1.2) = 2(g\sin30^o)1.2$ as appropriate |
| | $v^2 = (-5x^2/8 + 1.6)g$ or $v^2 = (y - 5y^2/8 + 1.2)g$ or $v^2 = (5z/2 - 5z^2/8 - 0.9)g$ | A1 | | |
| | (ii) [$v_{max}^2 = 1.6g$ or $0.8g - 0.4g + 1.2g$ or $5g - 2.5g - 0.9g$] | M1 | | For using $v_{max}^2 = v^2(0)$ or $v^2(0.8)$ or $v^2(2)$ as appropriate |
| | Maximum speed is $3.96ms^{-1}$ | A1 | | |
| | (iii) [$5x^2 - 12.8 = 0 \Rightarrow x = 1.6$, $5y^2 - 8y - 9.6 = 0 \Rightarrow y = 2.4$, $5z^2 - 20z + 7.2 = 0 \Rightarrow z = 3.6$] | M1 | | For solving v = 0 |
| | Displacement is 3.6m | A1 | 8 | |
| Alternative for parts (ii) and (iii) for candidates who use Newton's second law and SHM analysis. | | | | |
| | [$m\ddot{x} = mg\sin30^o - 0.75mg(0.8 + x)/1.2 \Rightarrow \ddot{x} = -\omega^2x$; $v^2 = \omega^2(a^2 - x^2)$] | M1 | | For using N2 with $v^2 = \omega^2(a^2 - x^2)$ |
| | $v^2 = 5g(a^2 - x^2)/8$ | A1 | | |
| | | M1 | | For using $v^2(-0.8) = 2(g\sin30^o)1.2$ |
| | $v^2 = 5g(2.56 - x^2)/8$ | A1 | | |
| | (ii) [ $v_{max}^2 = 5g \times 2.56 \div 8$] | M1 | | For using $v_{max}^2 = v^2(0)$ |
| | Maximum speed is $3.96ms^{-1}$ | A1 | | |
| | (iii) [$2.56 - x^2 = 0 \Rightarrow x = 1.6$] | M1 | | For solving v = 0 |
| | Displacement is 3.6m | A1 | | |

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| **6** | (i) [ ½m$7^2$ = ½m$v^2$ + 2mg] | M1 | | For using the principle of conservation of energy |
| | Speed is 3.13ms$^{-1}$ | A1 | | |
| | [T = m$v^2$/r] | M1 | | For using Newton's second law horizontally and a = $v^2$/r |
| | Tension is 1.96N | A1ft | 4 | |
| | (ii) [T – mgcos$\theta$ = m$v^2$/r] | M1 | | For using Newton's second law radially |
| | | M1 | | For using T = 0 (may be implied) |
| | $v^2$ = -2gcos$\theta$ | A1 | | |
| | | M1 | | For using the principle of conservation of energy |
| | ½m$7^2$ = ½m$v^2$ +mg(2 – 2cos$\theta$) | A1 | | |
| | [-2gcos$\theta$ = 49 –4g + 4gcos$\theta$] | M1 | | For eliminating $v^2$ |
| | 6gcos$\theta$ = -9.8 | A1 | | May be implied by answer |
| | $\theta$ = 99.6 | A1 | 8 | |
| Alternative for candidates who eliminate $v^2$ before using T = 0. | | | | |
| | (ii) [T – mgcos$\theta$ = m$v^2$/r] | M1 | | For using Newton's second law radially |
| | | M1 | | For using the principle of conservation of energy |
| | ½m$7^2$ = ½m$v^2$ +mg(2 – 2cos$\theta$) | A1 | | |
| | [T - mgcos$\theta$ = m(49 –4g + 4gcos$\theta$)2] | M1 | | For eliminating $v^2$ |
| | | M1 | | For using T = 0 (may be implied) |
| | -2g cos$\theta$ = 49 –4g + 4gcos$\theta$ | A1ft | | ft error in energy equation |
| | 6gcos$\theta$ = -9.8 | A1 | | May be implied by answer |
| | $\theta$ = 99.6 | A1 | 8 | |

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|---|---|---|---|---|
| **7** | (i) T = 4mg(4 + x – 3.2)/3.2 | B1 | | |
| | [ma = mg – 4mg(0.8 + x)/3.2] | M1 | | For using Newton's second law |
| | 4$\ddot{x}$ = -49x | A1 | 3 | AG |
| | (ii) Amplitude is 0.8m | B1 | | (from 4 + A = 4.8) |
| | Period is $2\pi/\omega$ s where $\omega^2$ = 49/4 | B1 | | |
| | | M1 | | String is instantaneously slack when shortest (4 - A = 3.2 = L). Thus required interval length = period. |
| | Slack at intervals of 1.8s | A1 | 4 | AG |
| | (iii) [ma = -mgsin$\theta$] | M1 | | For using Newton's second law tangentially |
| | mL$\ddot{\theta}$ = -mgsin$\theta$ | A1 | | |
| | For using sin$\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ –(g/L)$\theta$ | A1 | 3 | AG |
| | (iv) [$\theta$ = 0.08cos(3.5x0.25)] (= 0.05127..) | M1 | | For using = $_o$cosωt where $\omega^2$=12.25 (may be implied by $\dot{\vartheta}$ = -ω $_o$sinωt ) |
| | [$\dot{\theta}$ = -3.5(0.08)sin(3.5x0.25), $\dot{\theta}^2$ = 12.25($0.08^2$ – $0.05127..^2$)] | M1 | | For differentiating = $_o$cosωt and using $\dot{\vartheta}$ or for using $\dot{\theta}^2 = \omega^2(\theta_o{}^2 - \theta^2)$ where $\omega^2$=12.25 |
| | $\dot{\theta} = \mp$ 0.215 | A1 | | May be implied by final answer |
| | [v = 0.215x9.8/12.25] | M1 | | For using v = L$\dot{\vartheta}$ and L =g/$\omega^2$ |
| | Speed is 0.172 ms$^{-1}$ | A1 | 5 | |